8.311 Recitation Notes

Eric R. Anschuetz

MIT Center for Theoretical Physics,

77 Massachusetts Avenue,

Cambridge,

MA 02142,

*USA**

(Dated: April 25, 2019)

I. INTRODUCTION

Today I am going to go over common mistakes with index summation from the previous problem set.

II. REUSING INDICES

Consider the summations:

$$A_{\mu\nu} = B_{\mu\alpha} C_{\nu\beta} D^{\alpha} E^{\beta}, \tag{1}$$

$$A'_{\mu\nu} = B_{\mu\alpha} C_{\nu\alpha} D^{\alpha} E^{\alpha}. \tag{2}$$

Are A and A' equal? The answer is *no*. Expanded out:

$$A_{\mu\nu} = B_{\mu 0} C_{\nu\beta} D^{0} E^{\beta} + B_{\mu 1} C_{\nu\beta} D^{1} E^{\beta} + B_{\mu 2} C_{\nu\beta} D^{2} E^{\beta} + B_{\mu 3} C_{\nu\beta} D^{3} E^{\beta}, \tag{3}$$

$$A'_{\mu\nu} = B_{\mu 0} C_{\nu 0} D^{0} E^{0} + B_{\mu 1} C_{\nu 1} D^{1} E^{1} + B_{\mu 2} C_{\nu 2} D^{2} E^{2} + B_{\mu 3} C_{\nu 3} D^{3} E^{3}. \tag{4}$$

These are *different*.

III. MIXING CONTRAVARIANT AND COVARIANT

Often in the problem set I saw expressions of the form:

$$B = A^{\mu} A^{\mu}. \tag{5}$$

This is almost assuredly wrong. Not only mathematically in the sense that you are taking the interior product of two contravariant vectors, but you will also get incorrect answers. For instance, the square of the proper time element $\mathrm{d}\tau^2$ (which is invariant) is given by:

$$\mathrm{d}\tau^2 = x_{\mu} x^{\mu} = \mathrm{c}^2\,\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \tag{6}$$

and is therefore a scalar. However,

$$x^{\mu} x^{\mu} = \mathrm{c}^2\,\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \tag{7}$$

is *not* invariant under Lorentz transformations, and therefore is not a scalar.

* eans@mit.edu

IV. PROVING OBJECTS ARE SCALARS

Just because an object has no free indices does *not* mean it is a scalar. For instance, the time coordinate t has no free indices but is not a scalar; instead, it is a component of the four-vector x^μ. Thus, when proving objects are scalars you must show that they are invariant under Lorentz transformations.