8.311 Recitation Notes

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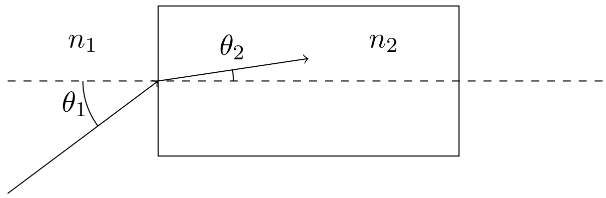

FIG. 1 A visual demonstration of Snell's law.

I. INTRODUCTION

Today I will talk about Fermat's principle.

II. FERMAT'S PRINCIPLE AND SNELL'S LAW

Huygen's principle states that each point on a wavefront acts as a fresh source of distribution of light. What does this look like in the ray optics picture?

It turns out that this is equivalent to *Fermat's principle*, which states that the path a ray of light takes between two points is the path of shortest time. Consider, for instance, the Snell's law setup of Fig. 1. Fixing the horizontal positions of the end points of the rays, what vertical position of the source (i.e. which $\tan(\theta_1)$) gives the shortest time length? We are thus minimizing:

$$t(\theta_1) = \frac{n_1}{\mathrm{c}} \cos(\theta_1) + \frac{n_2}{\mathrm{c}} \cos(\Theta - \theta_1), \tag{1}$$

where $\Theta = \theta_1 + \theta_2$ is fixed due to the fixed relative positions of the endpoints of the rays.

As:

$$\frac{\mathrm{d}t(\theta_1)}{\mathrm{d}\theta_1} = -\frac{n_1}{\mathrm{c}} \sin(\theta_1) + \frac{n_2}{\mathrm{c}} \sin(\Theta - \theta_1), \tag{2}$$

we therefore have that:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \tag{3}$$

and we have recovered Snell's law.

III. WHY FERMAT'S PRINCIPLE?

Fermat's principle follows the same general formulation as any principle of least action in classical mechanics, or the path integral formulation of quantum mechanics. How are these

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principles interpreted?

In the case of Fermat's principle applied to two points, we can think of light traveling along *all* possible paths between the two points. Because the amount of time taken between the two points is different depending on the path, the light acquires a different phase depending on the path taken. Furthermore, as $\frac{\mathrm{d}t(\theta_1)}{\mathrm{d}\theta_1} = 0$ for the extremal path, small deviations from this path will give only second order corrections to the phase. However, for a generic path, large first order deviations dominate and average to zero. Thus, the only constructive interference occurs at the path of least time, and this is what we see.