8.311 Recitation Notes

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I. INTRODUCTION

Today I will review some of the material that could be on your third exam. The exam will cover the same material covered in problem sets eight through ten.

II. MAXWELL'S EQUATIONS IN MATERIALS

In materials, Maxwell's equations are:

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho_{\mathrm{f}}, \tag{1}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{2}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{3}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J}_{\mathrm{f}} + \frac{\partial \boldsymbol{D}}{\partial t}, \tag{4}$$

where:

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}, \tag{5}$$

$$\boldsymbol{H} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M}, \tag{6}$$

and the subscript f denotes free charges and currents. Considering the bound charges and currents:

$$\rho_{\mathrm{b}} = -\boldsymbol{\nabla} \cdot \boldsymbol{P}, \tag{7}$$

$$\boldsymbol{J}_{\mathrm{b}} = \boldsymbol{\nabla} \times \boldsymbol{M} + \frac{\partial \boldsymbol{P}}{\partial t}, \tag{8}$$

these macroscopic Maxwell's equations reproduce the traditional Maxwell's equations, with:

$$\rho = \rho_{\mathrm{f}} + \rho_{\mathrm{b}}, \tag{9}$$

$$\boldsymbol{J} = \boldsymbol{J}_{\mathrm{f}} + \boldsymbol{J}_{\mathrm{b}}. \tag{10}$$

Often, we consider linear media, where we take:

$$\boldsymbol{D} = \epsilon \boldsymbol{E}, \tag{11}$$

$$\boldsymbol{H} = \frac{1}{\mu}\boldsymbol{B}. \tag{12}$$

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III. DISPERSION RELATIONS

As proven in the eleventh recitation notes, the general dispersion relation for a medium is:

$$\det\left(\boldsymbol{k}\otimes\boldsymbol{k}-k^2+\frac{\omega^2}{\epsilon_0 \mathrm{c}}\boldsymbol{\epsilon}\right)=0. \tag{13}$$

As:

$$\boldsymbol{\epsilon}=\epsilon_0+\frac{\mathrm{i}\boldsymbol{\sigma}}{\omega}, \tag{14}$$

this is equivalent to the dispersion relation:

$$\det\left(\boldsymbol{k}\otimes\boldsymbol{k}-k^2+\frac{\omega^2}{\mathrm{c}^2}+\mathrm{i}\mu_0\omega\boldsymbol{\sigma}\right)=0. \tag{15}$$

IV. SNELL'S LAW

Snell's law is the statement that, for a ray of light leaving a medium with an index of refraction n_1 and entering a medium with an index of refraction n_2, the angles of refraction when taken to the normal are related by:

$$n_1\sin(\theta_1)=n_2\sin(\theta_2). \tag{16}$$

V. BREWSTER'S ANGLE

Brewster's angle θ_B is a critical angle such that unpolarized light is reflected back perfectly polarized; it is given by:

$$\theta_\mathrm{B}=\arctan\left(\frac{n_2}{n_1}\right). \tag{17}$$

VI. REFLECTION AND TRANSMISSION

For an electric field with normal polarization to the plane of incidence leaving a medium with an index of refraction n_1 and entering a medium with an index of refraction n_2 at zero angle of incidence, the Fresnel equations give that the amplitude reflection and transmission coefficients are:

$$r=\frac{n_1-n_2}{n_1+n_2}, \tag{18}$$

$$t=\frac{2n_1}{n_1+n_2}. \tag{19}$$

VII. TIME AVERAGES OF COMPLEX FIELDS

When evaluating a time average of a product—say, $\langle \boldsymbol{J} \cdot \boldsymbol{B} \rangle$—what one should actually be evaluating is $\langle \mathrm{Re}\,(\boldsymbol{J}) \cdot \mathrm{Re}\,(\boldsymbol{B}) \rangle$, *not* $\langle \mathrm{Re}\,(\boldsymbol{J} \cdot \boldsymbol{B}) \rangle$. If one wants to take the real part at the end of the calculation, one must consider the fact that:

$$\langle \mathrm{Re}\,(\boldsymbol{J}) \cdot \mathrm{Re}\,(\boldsymbol{B}) \rangle = \frac{1}{2}\,\mathrm{Re}\,(\boldsymbol{J} \cdot \boldsymbol{B}^*)\,. \tag{20}$$

VIII. APPLICATIONS OF FARADAY'S LAW

The integral form of Faraday's law states that the electromotive force $\mathcal{E}$ around a loop is given by:

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}, \tag{21}$$

where Φ_B is the magnetic flux through the surface defined by the loop. For a battery with voltage V in series with a resistor of resistance R, the magnitude of the electromotive force between anode of the resistor and the anode of the battery is given by:

$$|\mathcal{E}| = |V - IR|\,, \tag{22}$$

where I is the current through the loop.